**Math 441/541 Homework Exercises 1** Name:________________________

**Question 1** Prove the following three claims mentioned in the book on page 7. Use the analogous results for open subsets of $\mathbb{R}^k$ on pages 3-4 of the text and mimic ideas in the proof on page 6 that for a smooth map $f : M \to N$ of smooth manifolds with $f(x) = y$, the map $Df_x : T_xM \to T_yN$ is a well-defined linear map.

a) (Chain Rule) If $f : M \to N$ and $g : N \to P$ are smooth, with $f(x) = y$, then $D(g \circ f)_x = Dg_y \circ Df_x$.

b) If $I$ is the identity map of $M$, then $DI_x$ is the identity map of $T_xM$. More generally, if $M \subset N$ with inclusion map $i$, then $T_xM \subset T_xN$ with inclusion map $Di_x$.

c) If $f : M \to N$ is a diffeomorphism, then $Df_x : T_xM \to T_yN$ is a vector space isomorphism. In particular, the dimension of $M$ is equal to the dimension of $N$.